PreCalculus – Summer Packet

Entering into PreCalculus means entering into your first year of college preparatory mathematics. There is a major shift in expectation for students to be able to recall many skills at a moment's notice. Certain concepts that have been taught to you over the previous years are assumed to be mastered. If you do not have these skills, you will find that you will consistently get problems incorrect next year as you make mistakes. It is frustrating for students who spend much of their homework time relearning algebra concepts in addition to learning how to tie the concepts together. This summer packet is intended for you to brush up and possibly relearn these topics.

On the following pages, you have assorted problems related to specific topics. Each problem should be done in the space provided. Rather than give you a textbook to remind you of the formulas and techniques necessary to solve the problem, there are a few websites listed that have full instructions on the techniques. If and when you are unsure of how to attempt these problems, use these websites. Don't fake your way through these problems. You are only setting yourself up for a future struggle.

Realize also that many concepts are interrelated. This will be the focus throughout next year as we examine the mathematical relationships between topics numerically, algebraically and graphically. While you may be strong in one of these approaches, you must learn to view each topic from the other two approaches as well to achieve full understanding. Success on your tests and quizzes throughout the year will depend on you being able to do so.

You need to get off to a good start so spend some quality time on this packet this summer. Tear off this first page and return the remaining sheets stapled together. Be sure your name appears on the first sheet. Work needs to be shown when needed. Also, do not rely on the calculator to work through the majority of these problems. You will be tested without the calculator, so practice without the calculator.

This packet is to be completed by the first day back in school in the fall. You will be tested over this material on the second day of class. You will also be expected to efficiently work through the problems under a time constraint. Many students are not prepared for this expectation and find they do not have the time to check their answers like they are used to. Prepare accordingly.

It is a mistake to decide to do this now. Let it go until mid-summer. We want these techniques to be relatively fresh in your mind in the fall. But, do not wait to do them at the very last minute. These take time. If you do a few concepts a day, the whole packet will take you about a week to complete.

We hope you take this seriously as we sincerely wish for you to be successful throughout this next year. Your preparation over the summer will be rewarded in unexpected ways during the year.

Good Luck!
We hope you have a wonderful summer!

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Answer key available on the EHS Gifted Academy or Mrs. Annable's website
www.ehsgiftedacademy.weebly.com www.annablesmath.weebly.com

Topics

Exponents

1. Basic Rules
2. Fractional Exponents
3. Simplifying Radicals

Factoring

4. Factoring by GCF
5. Factoring quadratic expressions
6. Special Factoring formulas
7. Factoring through synthetic division

Equations/Inequalities

8. Solving linear equations.
9. Solving quadratic equations by factoring
10. Solving quadratic equations by quadratic formula
11. Solving radical equations
12. Solving rational equations
13. Solving logarithmic equations

Functions

14. Function notation
15. Function names
16. Function Operations.

Graphing

17. Function transformations
18. Graphing Parent Functions by T-chart using “smart” points
19. Basic Graphing using “smart” points

General Topics

20. Distance and Midpoint formulas
21. Intercepts
22. Equations of lines
23. Pythagorean Theorem
24. Common Algebraic Errors

Name _____

1: Exponent Rules

Simplify the following

1. $(-2^2)^3$
2. $-\left(\frac{2}{5}\right)^{-2}$
3. $(3x^2y)^{-3}$
4. $\frac{y^{-4}}{5x^{-2}}$
5. $\frac{x^{-1}y}{xy^{-2}}$
6. $\frac{3xy^9}{2y^{-2}} \cdot \frac{-7y}{42x^5}$

2: Fractional Exponents

Evaluate the following without a calculator

1. $8^{\frac{2}{3}}$
2. $4^{-\frac{1}{2}}$
3. $\left(\sqrt[4]{16}\right)^2$
4. $\sqrt[3]{1000^2}$
5. $\left(\sqrt[3]{-27}\right)^{-4}$
6. $-(25^{-\frac{3}{2}})$

3: Simplifying Radicals

Simplify and rationalize the following.

1. $\sqrt{80}$
2. $\sqrt[4]{32}$
3. $\sqrt[3]{54x^3}$
4. $\frac{3}{\sqrt{8}}$
5. $\sqrt{\frac{4}{75}}$
6. $4\sqrt{3} \cdot \sqrt{21}$

4: Factoring by GCF

Factor the following completely

1. $3x^4 - 9x^2$
2. $49xy + 28x - 14y$
3. $18x^3y^5 - 12x^4y^2$

5: Factoring Quadratic Expressions

Factor the following completely

1. $x^2 - 3x + 2$
2. $x^2 + 5x - 6$
3. $2x^2 + 5x - 3$
4. $3x^2 - 8x + 4$
5. $3x^2 + 17x + 10$
6. $10x^2 - 19x + 6$

6: Special Factoring

$$a^2 + 2ab + b^2 = (a+b)^2 \qquad a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^2 - 2ab + b^2 = (a-b)^2 \qquad a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$a^2 - b^2 = (a+b)(a-b)$$

Factor the following completely

1. $4x^2 - 20x + 25$
2. $49x^2 + 42xy + 9y^2$
3. $16x^4 - 81$
4. $x^3 - 8$
5. $125x^3 + y^3$
6. $64 - 27y^6$

7: Factoring through Synthetic Division

Use synthetic division to factor as indicated.

1. $x^3 - 4x^2 + 2x + 1 = (x - 1)(\quad)$

2. $2x^3 + 5x + 7 = (x + 1)(\quad)$

3. $x^4 - 3x^3 + x^2 + x + 2 = (x - 2)(\quad)$

4. $4x^4 + 3x^2 - 1 = (2x - 1)(\quad)(\quad)$

8: Solving Linear Equations

Solve the following for the unknown variable.

1. $\frac{2x+1}{5} = \frac{3x+1}{2}$

2. $\frac{x}{2} + \frac{5x}{6} = \frac{2x}{3} + \frac{1}{12}$

3. $3(x - 8) + 4x = 5x - (x + 7)$

9: Solving Quadratic Equations by Factoring

Factor to solve for x.

1. $x^2 + 5x + 6 = 0$

2. $8x^2 - 6x - 5 = 0$

3. $11x^2 - 14x - 16 = 0$

10: Solving Quadratic Equations using the Quadratic Formula

For each equation, solve for the indicated expression.

1. $2x^2 - 4x - 1 = 0 \quad for\ x$
2. $2x^2 + 2x + 3 = 0 \quad for\ x$
3. $x^4 - 4x^2 + 2 = 0 \quad for\ x^2$

11: Solving Radical Equations

Solve the following for x.

1. $\sqrt{x} = 3x - 1$
2. $3\sqrt{2x} + 1 = 7$
3. $3x^{\frac{3}{4}} - 5 = 19$

12: Solving Rational Equations

Solve the following for x

1. $\frac{3}{2x} - \frac{9}{2} = 6x$
2. $\frac{2}{3x} + \frac{2}{3} = \frac{8}{x+6}$
3. $\frac{2}{x+1} + \frac{x}{x-1} = \frac{2}{x^2-1}$

13: Solving Logarithmic Equations

Solve the following for x

1. $\log_3 3^x = 7$
2. $\log_9 x = \frac{1}{2}$
3. $2\log_3(x+1) = 4$

14: Function Notation

Given $f(x) = -x^2 + x$, answer the following questions.

1. Find $f(0)$
2. Find $f(x) = 0$
3. Find $f\left(-\frac{1}{3}\right)$

Given $f(x) = \frac{1}{3}x + \frac{7}{4}$, answer the following questions.

4. Find the zeros of $f(x)$
5. Solve $f(x) = \frac{1}{8}$
6. Find $f\left(-\frac{9}{8}\right)$

15: Function Names

Match the following equations to their description.

_____1.	$f(x) = \frac{2}{3}\lvert 4x+5 \rvert - 3$	A.	Linear Function
_____2.	$f(x) = \frac{2}{3}\sqrt[3]{4x+5} - 3$	B.	Quadratic Function
_____3.	$f(x) = \frac{2}{3} \cdot \frac{1}{4x+5} - 3$	C.	Absolute Value Function
_____4.	$f(x) = \frac{2}{3}(4x+5)^4 - 3(4x+5)^2 - 2$	D.	Cubic Function
_____5.	$f(x) = \frac{2}{3}(4x+5)^3 - 3$	E.	Cube Root Function
_____6.	$f(x) = \frac{2}{3}(4x+5) - 3$	F.	Square Root Function
_____7.	$f(x) = \frac{2}{3}(4x+5)^2 - 3$	G.	Rational Function
_____8.	$f(x) = \frac{2}{3}\sqrt{4x+5} - 3$	H.	Polynomial Function

16: Function Operations

Perform the following function operations if $f(x)=2x^2$ and $g(x)=3-4x$

1. $f(g(x))$

2. $g(f(x))$

3. $(f-g)(x)$

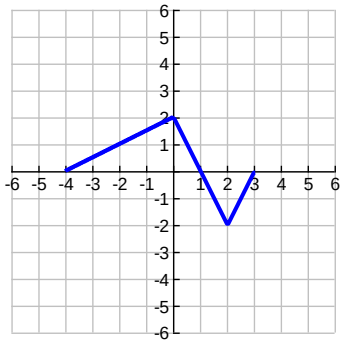
4. $f(f(x))$

5. $g(g(x))$

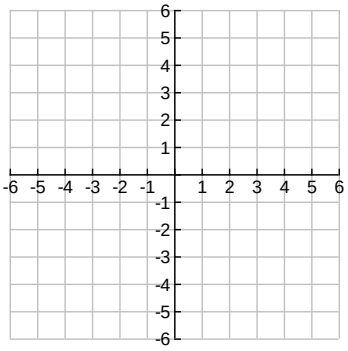
6. Find $g(g(x))=0$

17: Function Transformation

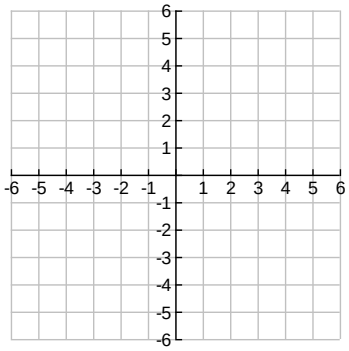
Use the graph of $y=f(x)$ at the right to sketch the following transformations.



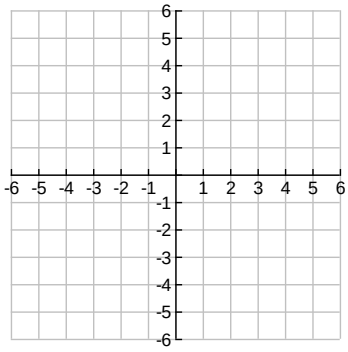
1. $y=2f(x)$



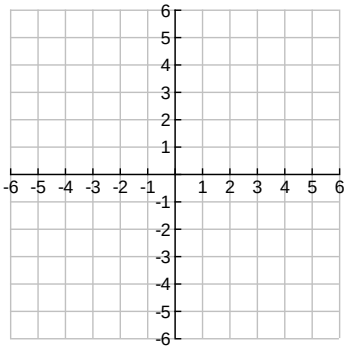
2. $y=-f(x)$



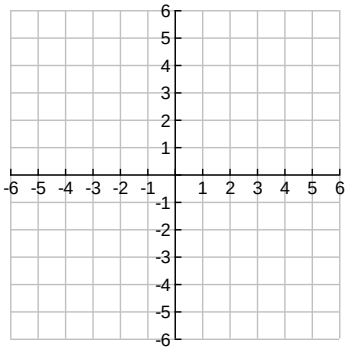
3. $y=f(x-1)$



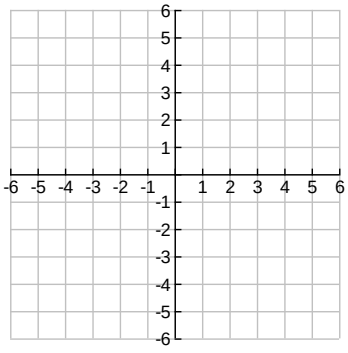
4. $y=f(x)+2$



5. $y=f(-x)$



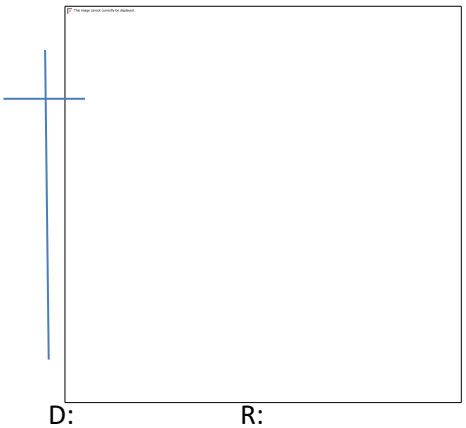
6. $y=-2f(x+2)+1$



18: Graphing Parent Functions using T-Charts

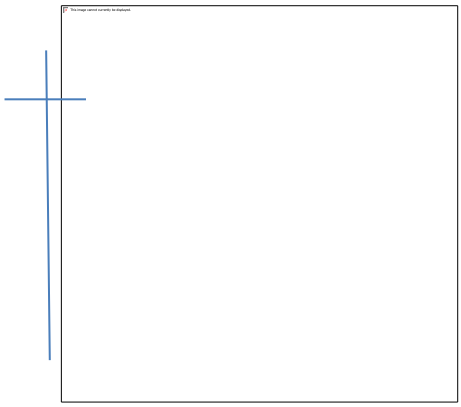
Graph the following using a T-Chart with "smart" values. State the Domain and Range of each function.

1. $f(x) = x^2$



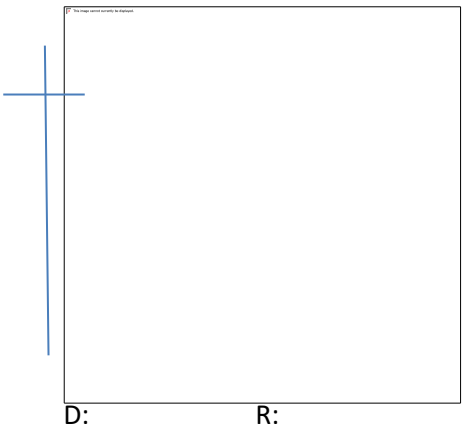
D: R:

2. $f(x) = \sqrt{x}$



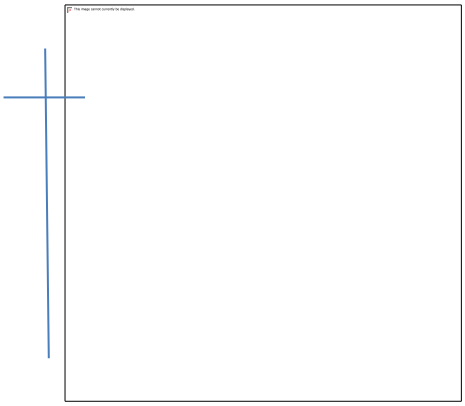
D: R:

3. $f(x) = |x|$



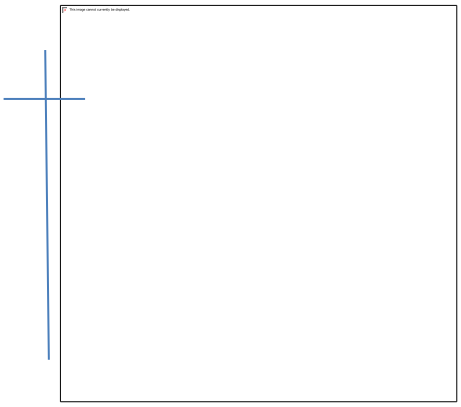
D: R:

4. $f(x) = x^3$



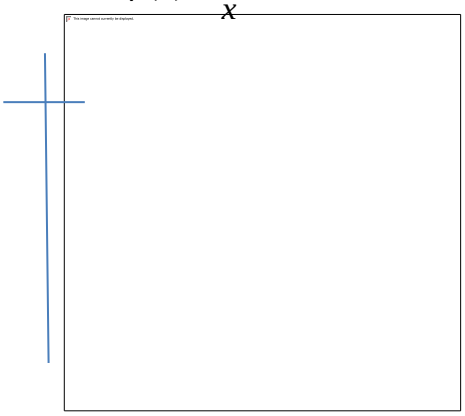
D: R:

5. $f(x) = \sqrt[3]{x}$



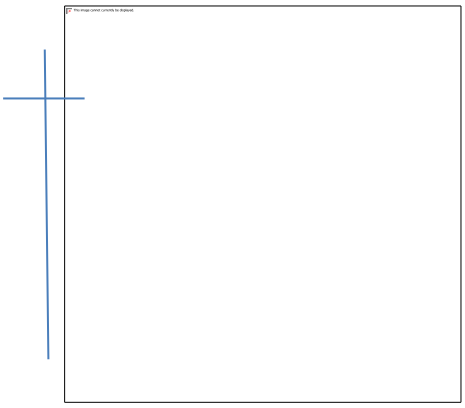
D: R:

6. $f(x) = \frac{1}{x}$



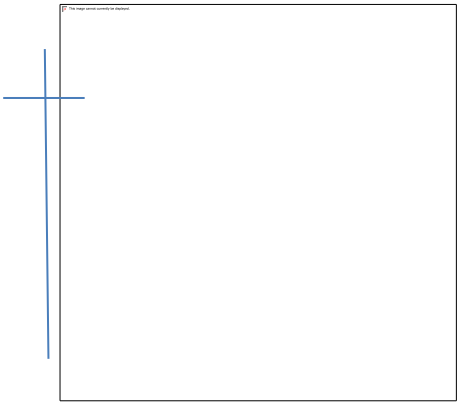
D: R:

7. $f(x) = x$



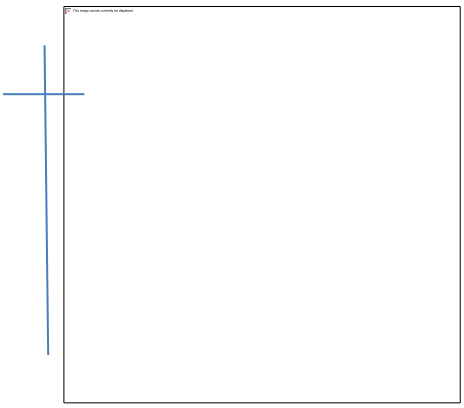
D: R:

8. $f(x) = 2^x$



D: R:

9. $f(x) = \log_2 x$



D: R:

19: Basic Graphing Choosing "Smart" Points

Fill in the T-chart using at least 3 smart x-values (that enable you to find exact points)

1. $f(x)=\sqrt{3-x}$

2. $f(x)=\dfrac{7}{x-2}$

3. $f(x)=3^{\frac{x}{4}}$

20: Distance and Midpoint Formulas

Find the distance between the two points. Then find the midpoint between the two points.

1. $(-2,5);\ (6,-1)$

2. $\left(\dfrac{3}{2},-\dfrac{1}{2}\right);\ \left(-\dfrac{3}{2},\dfrac{7}{2}\right)$

3. $\left(\dfrac{5}{2},-\dfrac{3}{2}\right);\ (1,-4)$

21: Intercepts

Use the following equations to find the x and y intercept(s)

1. $y^2=x+9$

2. $9x^2+4y^2=36$

3. $\left(\dfrac{x+4}{2}\right)^2+y^2=1$

22: Equations of Lines

Find the equation of the line that has the given characteristics. Leave your answer in the form indicated.

1. $slope=\dfrac{3}{4};\ y-\text{int}:-\dfrac{2}{3}$

(Standard Form)

2. Parallel to $2x+3y=4$ through $(-3,6)$

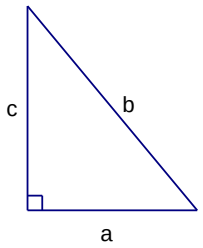
(Slope-intercept form)

3. Perpendicular to $4x-7y=23$ through $\left(\dfrac{2}{3},-\dfrac{4}{5}\right)$

(Point-Slope Form)

23: Pythagorean Theorem

Use the diagram at the right to answer the following questions. Be sure to simplify.



1. Find b if a = $4\sqrt{5}$, c = 2

2. Find c if a = $2\sqrt{3}$, b=6

3. If a = c, and b = 10, find a

24: Algebraic Errors to Avoid

Error	*Correct form*	*Comments*
$a - (x - b) \neq a - x - b$	$a - (x - b) = a - x + b$	Change all signs when distribution negative through parentheses.
$(a + b)^2 \neq a^2 + b^2$	$(a + b)^2 = a^2 + 2ab + b^2$	Don't forget middle term when squaring binomials.
$\left(\frac{1}{2}a\right)\left(\frac{1}{2}b\right) \neq \frac{1}{2}ab$	$\left(\frac{1}{2}a\right)\left(\frac{1}{2}b\right) = \frac{1}{4}(ab)$	1/2 occurs twice as a factor.
$\frac{a}{x + b} \neq \frac{a}{x} + \frac{a}{b}$	Leave as $\frac{a}{x + b}$	Don't add denominators when adding fractions.
$\frac{1}{a} + \frac{1}{b} \neq \frac{1}{a + b}$	$\frac{1}{a} + \frac{1}{b} = \frac{a + b}{ab}$	Use definition for adding fractions.
$\frac{\frac{x}{a}}{b} \neq \frac{bx}{a}$	$\frac{\frac{x}{a}}{b} = \left(\frac{x}{a}\right)\left(\frac{1}{b}\right) = \frac{x}{ab}$	Multiply by reciprocal of the denominator.
$\frac{1}{3x} \neq \frac{1}{3}x$	$\frac{1}{3x} = \frac{1}{3} \cdot \frac{1}{x}$	Use definition for multiplying fractions.
$1/x + 2 \neq \frac{1}{x + 2}$	$1/x + 2 = \frac{1}{x} + 2$	Be careful when using a slash to denote division.
$(x^2)^3 \neq x^5$	$(x^2)^3 = x^{2 \cdot 3} = x^6$	Multiply exponents when an exponential form is raised to a power.
$2x^3 \neq (2x)^3$	$2x^3 = 2(x^3)$	Exponents have priority over coefficients.
$\frac{1}{x^2 + x^3} \neq x^{-2} + x^{-3}$	Leave as $\frac{1}{x^2 + x^3}$	Don't shift term-by-term from denominator to numerator.
$\sqrt{5x} \neq 5\sqrt{x}$	$\sqrt{5x} = \sqrt{5}\sqrt{x}$	Radicals apply to every factor inside radical.
$\sqrt{x^2 + a^2} \neq x + a$	Leave as $\sqrt{x^2 + a^2}$	Don't apply radicals term-by-term.
$\frac{a + bx}{a} \neq 1 + bx$	$\frac{a + bx}{a} = 1 + \frac{b}{a}x$	Cancel common factor, *not* common terms.
$\frac{a + ax}{a} \neq a + x$	$\frac{a + ax}{a} = 1 + x$	Factor *before* canceling.