

Name _____

KEY

1: Exponent Rules

Simplify the following

1. $(-2^2)^3$

$$\boxed{-64}$$

2. $-\left(\frac{2}{5}\right)^{-2}$

$$\boxed{-\frac{25}{4}}$$

3. $(3x^2y)^{-3}$

$$\boxed{\frac{1}{27x^6y^3}}$$

4. $\frac{y^{-4}}{5x^{-2}}$

$$\boxed{\frac{x^2}{5y^4}}$$

5. $\frac{x^{-1}y}{xy^{-2}}$

$$\boxed{\frac{y^3}{x^2}}$$

6. $\frac{3xy^9}{2y^{-2}} \cdot \frac{-7y}{42x^5}$

$$\boxed{-\frac{y^{12}}{4x^4}}$$

2: Fractional Exponents

Evaluate the following without a calculator

1. $8^{\frac{2}{3}}$

$$\boxed{4}$$

2. $4^{\frac{1}{2}}$

$$\boxed{\frac{1}{2}}$$

3. $(\sqrt{16})^2$

$$\boxed{4}$$

4. $\sqrt[3]{1000^2}$

$$\boxed{100}$$

5. $(\sqrt[3]{-27})^{-4}$

$$\boxed{\frac{1}{81}}$$

6. $-(25^{\frac{3}{2}})$

$$\boxed{-\frac{1}{125}}$$

3: Simplifying Radicals

Simplify and rationalize the following.

1. $\sqrt{80}$

$$\boxed{4\sqrt{5}}$$

2. $\sqrt[4]{32}$

$$\boxed{2\sqrt[4]{2}}$$

3. $\sqrt[3]{54x^3}$

$$\boxed{3x\sqrt[3]{2}}$$

4. $\frac{3}{\sqrt{8}} \cdot \frac{\sqrt{8}}{\sqrt{8}}$

$$\frac{3\sqrt{8}}{8}$$

$$\frac{6\sqrt{2}}{8} = \boxed{\frac{3\sqrt{2}}{4}}$$

5. $\sqrt{\frac{4}{75}}$

$$\frac{2}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{\frac{2\sqrt{3}}{15}}$$

6. $4\sqrt{3} \cdot \sqrt{21}$

$$4\sqrt{63}$$

$$\boxed{12\sqrt{7}}$$

4: Factoring by GCF

Factor the following completely

1. $3x^4 - 9x^2$

$$3x^2(x^2 - 3)$$

2. $49xy + 28x - 14y$

$$7(7xy + 4x - 2y)$$

3. $18x^3y^5 - 12x^4y^2$

$$6x^3y^2(3y^3 - 2x)$$

5: Factoring Quadratic Expressions

Factor the following completely

1. $x^2 - 3x + 2$

$$(x-2)(x-1)$$

2. $x^2 + 5x - 6$

$$(x+6)(x-1)$$

3. $2x^2 + 5x - 3$

$$(2x-1)(x+3)$$

4. $3x^2 - 8x + 4$

$$\begin{array}{r} 12 \\ -6 \times -2 \\ -8 \end{array} (3x-2)(x-2)$$

5. $3x^2 + 17x + 10$

$$\begin{array}{r} 30 \\ 15 \times 2 \\ 17 \end{array} (3x+2)(x+5)$$

6. $10x^2 - 19x + 6$

$$\begin{array}{r} 60 \\ -15 \times -4 \\ -19 \end{array} (5x-2)(2x-3)$$

6: Special Factoring

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^2 - b^2 = (a+b)(a-b)$$

Factor the following completely

1. $4x^2 - 20x + 25$

$$(2x-5)^2$$

2. $49x^2 + 42xy + 9y^2$

$$(7x + 3y)^2$$

3. $16x^4 - 81$

$$(4x^2 - 9)(4x^2 + 9)$$

4. $x^3 - 8$

$$(x-2)(x^2 + 2x + 4)$$

5. $125x^3 + y^3$

$$(5x + y)(25x^2 - 5xy + y^2)$$

6. $64 - 27y^6$

$$(4 - 3y^2)(16 + 12y^2 + 9y^4)$$

7: Factoring through Synthetic Division

Use synthetic division to factor as indicated.

1. $x^3 - 4x^2 + 2x + 1 = (x-1)(\quad)$

$$\begin{array}{r|rrrr} -1 & 1 & -4 & 2 & 1 \\ & & 1 & -3 & -1 \\ \hline & 1 & -3 & -1 & 0 \end{array}$$

$$(x-1)(x^2 - 3x - 1)$$

2. $2x^3 + 5x + 7 = (x+1)(\quad)$

$$\begin{array}{r|rrrr} -1 & 2 & 0 & 5 & 7 \\ & & -2 & 2 & -7 \\ \hline & 2 & -2 & 7 & 0 \end{array}$$

$$(x+1)(2x^2 - 2x + 7)$$

3. $x^4 - 3x^3 + x^2 + x + 2 = (x-2)(\quad)$

$$\begin{array}{r|rrrrr} 2 & 1 & -3 & 1 & 1 & 2 \\ & & 2 & -2 & -2 & -2 \\ \hline & 1 & -1 & -1 & -1 & 0 \end{array}$$

$$(x-2)(x^3 - x^2 - x - 1)$$

4. $4x^4 + 3x^2 - 1 = (2x-1)(\quad)$

use long division for this one!

$$\begin{array}{r} 2x^3 + x^2 + 2x + 1 \\ 2x-1 \overline{) 4x^4 - 1} \\ \underline{-(4x^4 - 2x^3)} \\ 2x^3 + 3x^2 \\ \underline{-(2x^3 - x^2)} \\ 4x^2 \\ \underline{-(4x^2 - 2x)} \\ 2x - 1 \\ \underline{-(2x - 1)} \\ 0 \end{array}$$

$$(2x^3 + x^2 + 2x + 1)$$

8: Solving Linear Equations

Solve the following for the unknown variable.

1. $\frac{2x+1}{5} = \frac{3x+1}{2}$

$$4x + 2 = 15x + 5$$

$$-3 = 11x$$

$$x = \frac{-3}{11}$$

2. $\left(\frac{x}{2} + \frac{5x}{6} = \frac{2x}{3} + \frac{1}{12}\right) 12$

$$6x + 10x = 8x + 1$$

$$16x = 8x + 1$$

$$8x = 1$$

$$x = \frac{1}{8}$$

3. $3(x-8) + 4x = 5x - (x+7)$

$$3x - 24 + 4x = 5x - x - 7$$

$$7x - 24 = 4x - 7$$

$$3x = 17$$

$$x = \frac{17}{3}$$

9: Solving Quadratic Equations by Factoring

Factor to solve for x.

1. $x^2 + 5x + 6 = 0$

$$(x+3)(x+2) = 0$$

$$x = -3 \quad x = -2$$

2. $8x^2 - 6x - 5 = 0$

$$\begin{array}{r} -40 \\ -10 \times 4 \\ -6 \end{array} (4x-5)(2x+1)$$

$$4x-5=0 \quad 2x+1=0$$

$$x = \frac{5}{4} \quad x = -\frac{1}{2}$$

3. $11x^2 - 14x - 16 = 0$

$$\begin{array}{r} -176 \\ -22 \times 8 \\ -14 \end{array} (11x+8)(x-2)$$

$$11x+8=0 \quad x-2=0$$

$$x = \frac{-8}{11} \quad x = 2$$

10: Solving Quadratic Equations using the Quadratic Formula

For each equation, solve for the indicated expression.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. $2x^2 - 4x - 1 = 0$ for x

$$\frac{4 \pm \sqrt{16 - 4(2)(-1)}}{4} =$$

$$\frac{4 \pm \sqrt{20}}{4} = \frac{4 \pm 2\sqrt{5}}{4}$$

$$x = \frac{2 \pm \sqrt{5}}{2}$$

2. $2x^2 + 2x + 3 = 0$ for x

$$\frac{-2 \pm \sqrt{4 - 4(2)(3)}}{4} =$$

$$\frac{-2 \pm \sqrt{-20}}{4} = \frac{-2 \pm 2i\sqrt{5}}{4}$$

$$x = \frac{-1 \pm i\sqrt{5}}{2}$$

3. $x^4 - 4x^2 + 2 = 0$ for $x^2 = u$

$$\frac{4 \pm \sqrt{16 - 4(1)(2)}}{2} \quad u^2 - 4u + 2 = 0$$

$$\frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2}$$

$$x^2 = 2 \pm \sqrt{2}$$

11: Solving Radical Equations

Solve the following for x .

1. $(\sqrt{x} = 3x - 1)^2$

$$x = 9x^2 - 6x + 1$$

$$0 = 9x^2 - 7x + 1$$

$$\frac{7 \pm \sqrt{49 - 4(9)(1)}}{18}$$

$$\frac{7 \pm \sqrt{13}}{18} = .589$$

$$\frac{7 + \sqrt{13}}{18}$$

2. $3\sqrt{2x+1} = 7$

$$3\sqrt{2x+1} = 7$$

$$\sqrt{2x+1} = \frac{7}{3}$$

$$2x+1 = \frac{49}{9}$$

$$2x = \frac{40}{9}$$

$$x = \frac{20}{9}$$

3. $3x^{\frac{3}{4}} - 5 = 19$

$$3x^{\frac{3}{4}} = 24$$

$$(x^{\frac{3}{4}})^{\frac{4}{3}} = (8)^{\frac{4}{3}}$$

$$x = 16$$

12: Solving Rational Equations

Solve the following for x

1. $\left(\frac{3}{2x} - \frac{9}{2} = 6x\right) 2x$

$$3 - 9x = 12x^2$$

$$0 = 12x^2 + 9x - 3$$

$$0 = 3(4x^2 + 3x - 1)$$

$$3(4x-1)(x+1) = 0$$

$$x = \frac{1}{4}, -1$$

2. $\left(\frac{2}{3x} + \frac{2}{3} = \frac{8}{x+6}\right) 3x(x+6)$

$$2(x+6) + 2x(x+6) = 8(3x)$$

$$2x+12 + 2x^2+12x = 24x$$

$$2x^2 - 10x + 12 = 0$$

$$2(x^2 - 5x + 6) = 0$$

$$2(x-3)(x-2)$$

$$x = 3, 2$$

3. $\left(\frac{2}{x+1} + \frac{x}{x-1} = \frac{2}{x^2-1}\right) (x+1)(x-1)$

$$2(x-1) + x(x+1) = 2$$

$$2x-2 + x^2+x = 2$$

$$x^2 + 3x - 4 = 0$$

$$(x+4)(x-1)$$

$$x = -4, 1$$

$$x = -4$$

13: Solving Logarithmic Equations

Solve the following for x

1. $\log_3 3^x = 7$

$$3^7 = 3^x$$

$$x = 7$$

2. $\log_9 x = \frac{1}{2}$

$$9^{\frac{1}{2}} = x$$

$$x = 3$$

3. $2\log_3(x+1) = 4$

$$\log_3(x+1)^2 = 4$$

$$3^4 = (x+1)^2$$

$$3^2 = \pm(x+1)$$

$$8 = x$$

$$-10 = x$$

14: Function Notation

Given $f(x) = -x^2 + x$, answer the following questions.

1. Find $f(0)$

$$-0^2 + 0$$

$$\boxed{f(0) = 0}$$

2. ~~Find~~ ^{Solve} $f(x) = 0$

$$0 = -x^2 + x$$

$$-x(x-1)$$

$$\boxed{x = 0, 1}$$

3. Find $f\left(-\frac{1}{3}\right)$

$$-\left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)$$

$$-\frac{1}{9} - \frac{1}{3}$$

$$\boxed{-\frac{4}{9}}$$

Given $f(x) = \frac{1}{3}x + \frac{7}{4}$, answer the following questions.

4. Find the zeros of $f(x)$

$$0 = \frac{1}{3}x + \frac{7}{4}$$

$$-\frac{7}{4} = \frac{1}{3}x$$

$$\boxed{-\frac{21}{4} = x}$$

5. Solve $f(x) = \frac{1}{8}$

$$\frac{1}{8} = \frac{1}{3}x + \frac{7}{4}$$

$$-\frac{13}{8} = \frac{1}{3}x$$

$$\boxed{-\frac{39}{8} = x}$$

6. Find $f\left(-\frac{9}{8}\right)$

$$\frac{1}{3}\left(-\frac{9}{8}\right) + \frac{7}{4}$$

$$-\frac{3}{8} + \frac{7}{4}$$

$$\boxed{f\left(-\frac{9}{8}\right) = \frac{11}{8}}$$

15: Function Names

Match the following equations to their description.

C 1.

$$f(x) = \frac{2}{3}|4x+5| - 3$$

E 2.

$$f(x) = \frac{2}{3}\sqrt[3]{4x+5} - 3$$

G 3.

$$f(x) = \frac{2}{3} \cdot \frac{1}{4x+5} - 3$$

H 4.

$$f(x) = \frac{2}{3}(4x+5)^4 - 3(4x+5)^2 - 2$$

D 5.

$$f(x) = \frac{2}{3}(4x+5)^3 - 3$$

A 6.

$$f(x) = \frac{2}{3}(4x+5) - 3$$

B 7.

$$f(x) = \frac{2}{3}(4x+5)^2 - 3$$

F 8.

$$f(x) = \frac{2}{3}\sqrt{4x+5} - 3$$

A.

Linear Function

~~B.~~

Quadratic Function

~~C.~~

Absolute Value Function

~~D.~~

Cubic Function

~~E.~~

Cube Root Function

~~F.~~

Square Root Function

~~G.~~

Rational Function

~~H.~~

Polynomial Function

16: Function Operations

Perform the following function operations if $f(x) = 2x^2$ and $g(x) = 3 - 4x$

1. $f(g(x))$

$$2(3-4x)^2$$

$$2(9 - 24x + 16x^2)$$

$$32x^2 - 48x + 18$$

2. $g(f(x))$

$$3 - 4(2x^2)$$

$$3 - 8x^2$$

3. $(f-g)(x)$

$$2x^2 - (3 - 4x)$$

$$2x^2 - 3 + 4x$$

$$2x^2 + 4x - 3$$

4. $f(f(x))$

$$2(2x^2)^2$$

$$2(4x^4)$$

$$8x^4$$

5. $g(g(x))$

$$3 - 4(3 - 4x)$$

$$3 - 12 + 16x$$

$$16x - 9$$

6. Find $g(g(x)) = 0$

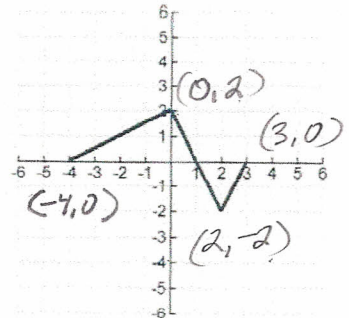
$$16x - 9 = 0$$

$$16x = 9$$

$$x = \frac{9}{16}$$

17: Function Transformation

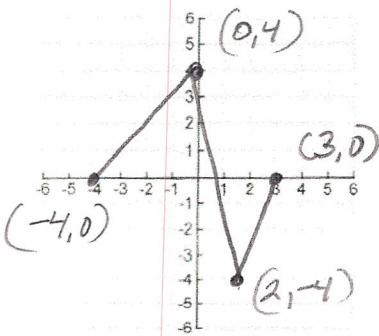
Use the graph of $y = f(x)$ at the right to sketch the following transformations.



(x, y)

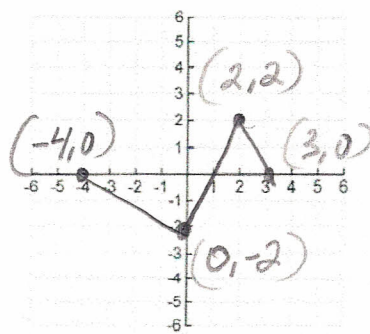
1. $y = 2f(x)$

$y * 2$



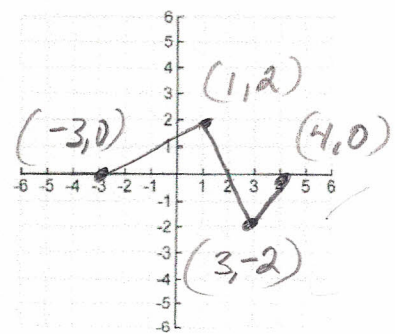
2. $y = -f(x)$

$y * -1$



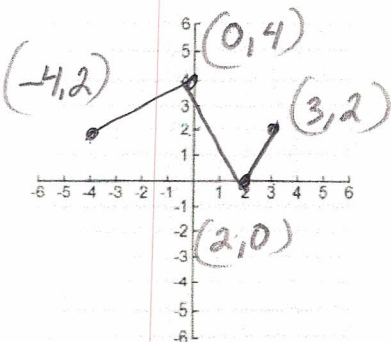
3. $y = f(x-1)$

$x + 1$



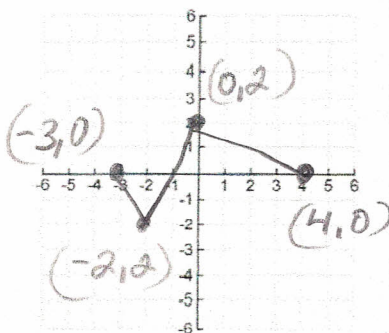
4. $y = f(x) + 2$

$y + 2$



5. $y = f(-x)$

$x * -1$

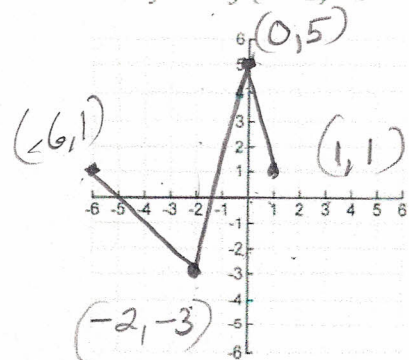


6. $y = -2f(x+2) + 1$

$x - 2$

$y * -2$

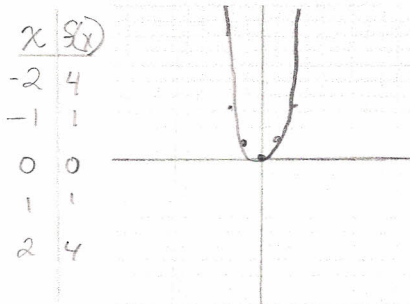
$+ 1$



18: Graphing Parent Functions using T-Charts

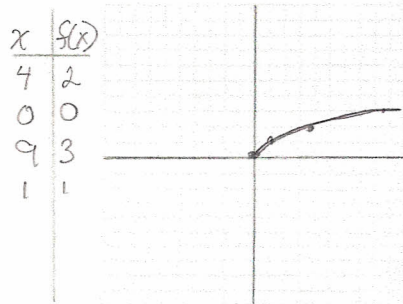
Graph the following using a T-Chart with "smart" values. State the Domain and Range of each function.

1. $f(x) = x^2$



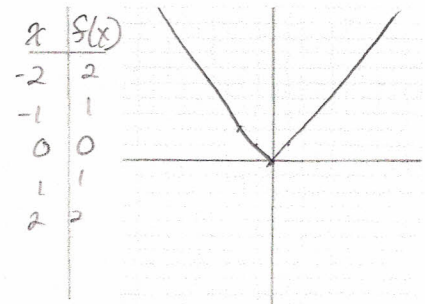
D: $(-\infty, \infty)$ R: $[0, \infty)$

2. $f(x) = \sqrt{x}$



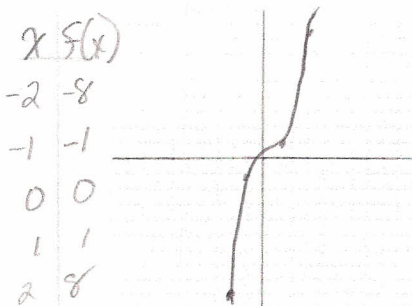
D: $[0, \infty)$ R: $[0, \infty)$

3. $f(x) = |x|$



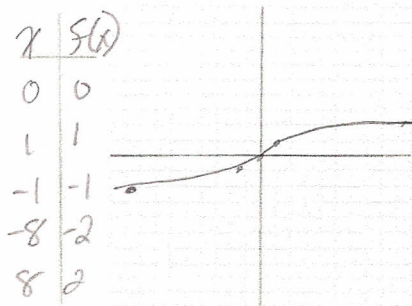
D: $(-\infty, \infty)$ R: $[0, \infty)$

4. $f(x) = x^3$



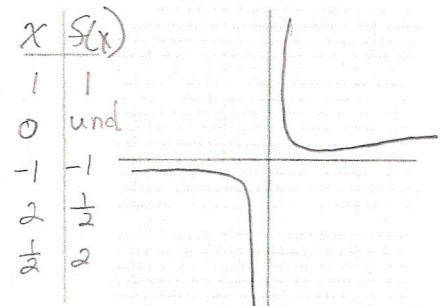
D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

5. $f(x) = \sqrt[3]{x}$



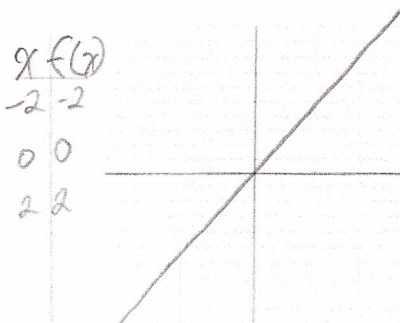
D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

6. $f(x) = \frac{1}{x}$



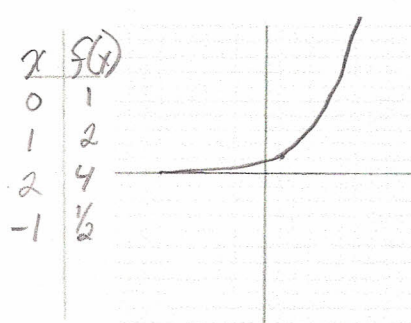
D: $x \neq 0$ R: $x \neq 0$

7. $f(x) = x$



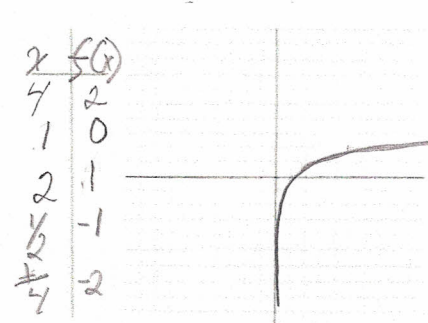
D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

8. $f(x) = 2^x$



D: $(-\infty, \infty)$ R: $(0, \infty)$

9. $f(x) = \log_2 x$



D: $(0, \infty)$ R: $(-\infty, \infty)$

19: Basic Graphing Choosing "Smart" Points

Fill in the T-chart using at least 3 smart x-values (that enable you to find exact points)

1. $f(x) = \sqrt{3-x}$

x	f(x)
3	0
-6	3
2	1
-1	2

2. $f(x) = \frac{7}{x-2}$

x	f(x)
3	7
1	-7
9	1
-5	-1

3. $f(x) = 3^{\frac{x}{4}}$

x	f(x)
0	1
4	3
8	9

20: Distance and Midpoint Formulas

Find the distance between the two points. Then find the midpoint between the two points.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$mid = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

1. $(-2, 5); (6, -1)$

$$D = \sqrt{(6 - (-2))^2 + (-1 - 5)^2}$$

$$= \sqrt{64 + 36} = \sqrt{100} = 10$$

2. $\left(\frac{3}{2}, -\frac{1}{2}\right); \left(-\frac{3}{2}, \frac{7}{2}\right)$

$$D = \sqrt{\left(-\frac{3}{2} - \frac{3}{2}\right)^2 + \left(\frac{7}{2} - \left(-\frac{1}{2}\right)\right)^2}$$

$$D = \sqrt{9 + 16} = 5$$

3. $\left(\frac{5}{2}, -\frac{3}{2}\right); (1, -4)$

$$D = \sqrt{\left(1 - \frac{5}{2}\right)^2 + \left(-4 - \left(-\frac{3}{2}\right)\right)^2}$$

$$= \sqrt{\frac{9}{4} + \frac{25}{4}} = \frac{\sqrt{34}}{2}$$

$$m = \left(\frac{-2 + 6}{2}, \frac{5 + (-1)}{2} \right) = (2, 2)$$

$$m = \left(\frac{\frac{3}{2} + \left(-\frac{3}{2}\right)}{2}, \frac{-\frac{1}{2} + \frac{7}{2}}{2} \right) = \left(0, \frac{3}{2}\right)$$

$$m = \left(\frac{\frac{5}{2} + 1}{2}, \frac{-\frac{3}{2} + (-4)}{2} \right) = \left(\frac{7}{4}, -\frac{11}{4}\right)$$

21: Intercepts

Use the following equations to find the x and y intercept(s)

1. $y^2 = x + 9$

$$0 = x + 9 \quad y^2 = 0 + 9$$

$$-9 = x \quad y^2 = 9$$

$$\quad \quad \quad y = \pm 3$$

$$(-9, 0)$$

$$(0, 3)$$

$$(0, -3)$$

2. $9x^2 + 4y^2 = 36$

$$9(0) + 4y^2 = 36 \quad 9x^2 + 4(0) = 36$$

$$y^2 = 9 \quad x^2 = 4$$

$$y = \pm 3 \quad x = \pm 2$$

$$(0, 3) \quad (2, 0)$$

$$(0, -3) \quad (-2, 0)$$

3. $\left(\frac{x+4}{2}\right)^2 + y^2 = 1$

$$\left(\frac{0+4}{2}\right)^2 + y^2 = 1$$

$$2 + y^2 = 1$$

$$y^2 = -1$$

NO y-int

$$(-2, 0)$$

$$(-6, 0)$$

22: Equations of Lines

Find the equation of the line that has the given characteristics. Leave your answer in the form indicated.

1. slope = $\frac{3}{4}$; y-int: $-\frac{2}{3}$

$$Ax + By = C$$

(Standard Form)

$$y = \frac{3}{4}x - \frac{2}{3}$$

$$12y = 9x - 8$$

$$9x - 12y = 8$$

2. Parallel to $2x + 3y = 4$ through

$$(-3, 6)$$

(Slope-intercept form) $y = mx + b$

$$3y = -2x + 4$$

$$y = -\frac{2}{3}x + \frac{4}{3}$$

$$y = -\frac{2}{3}x + b \quad (-3, 6)$$

$$6 = -\frac{2}{3}(-3) + b$$

$$6 = 2 + b$$

$$b = 4$$

$$y = -\frac{2}{3}x + 4$$

3. Perpendicular to $4x - 7y = 23$

through $\left(\frac{2}{3}, -\frac{4}{5}\right)$

(Point-Slope Form) $y - y_1 = m(x - x_1)$

$$-7y = -4x + 23$$

$$y = \frac{4}{7}x - \frac{23}{7}$$

$$y = -\frac{7}{4}x + b$$

$$y + \frac{4}{5} = -\frac{7}{4}\left(x - \frac{2}{3}\right)$$